FNC010,FNC013 - problem sheet 1

July 19, 2005

Instructions: answer all the questions. Marks will be awarded for calculation steps and demonstrable comprehension. The deadline for this assignment is the **26th of July**. *Please submit solutions in electronic format via email to wlwoon@must.edu.my and include "math-ps1" in the subject line. This is* ***important*** *- I filter my mails into separate folders and if you do not include that line your assignment may not get marked, resulting in a loss of marks*

Limits

1. Starting from the definition of limits, prove the following:

$$\lim_{x \to 1} 2x + 5 = 7$$
$$\lim_{x \to 2} (x + 3)(x - 1) = 5$$

2. Evaluate the following limits:

$$\lim_{x \to 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right)$$
$$\lim_{x \to 5} \frac{x^2 - 9}{x + 3}$$

Derivatives

1. From first principles, derive the derivatives (ahem) for the following expressions w.r.t x:

$$f(x) = x^3$$
$$f(x) = \cos x$$

2. (a) For the following, differentiate y w.r.t. x:

$$\begin{aligned} y &= (x^2 + \cos x^2) \\ y &= \frac{x^2 + 1}{x} \\ y &= \sin x^2 \cos 2x \\ y &= \cos u, u = \sin x \end{aligned}$$

(b) Differentiate $f(x) = x^3 + x$ w.r.t. x and sketch $f'(x)$.

Using this, sketch a figure of $f(x)$ for the region $-1 \leq x \leq 1$, explaining how you obtained the sketch. Finally, infer the minimum value for the gradient of this function.

3. Find a value of A which makes the function $f(x)$ continuous:

$$f(x) = \begin{cases} x^2 + 5 & \text{if } x < 1 \\ Ax - 5 & \text{if } x > 1 \end{cases}$$